UMBC

State Tables & Diagrams

CMSC 313

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Overview

- State Equation
- State Table
- State diagram

Current and Next state notation

	For 1 flip flop in equations	For 1 flip flop in table	For 1+ flip flop in equations	For 1+ flip flop in table
Current state	Q_n	Q	$Q_A(t)$, $Q_B(t)$	Q_A, Q_B
Next state	Q_{n+1}	Q+	$Q_A(t+1)$, $Q_B(t+1)$	Q_A+, Q_B+

State Equation

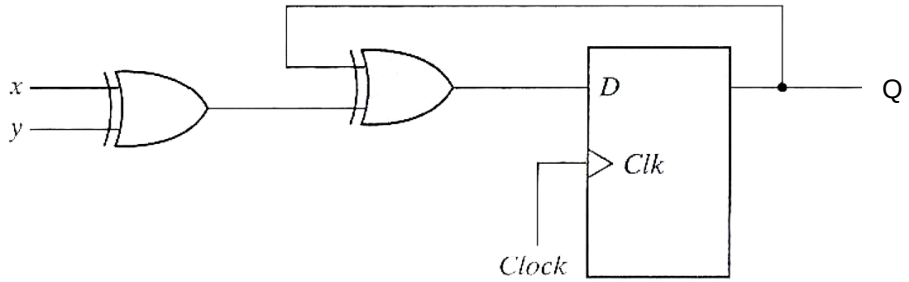
- A state equation is an algebraic expression that specifies the condition for a flip-flop state transition.
- The left side of the equation denotes the inputs to the flip-flop one clock edge later or Q(t+1)
- The right side of the equation is Boolean expression that specifies the present state and input conditions that make the next state equal to 1.
- Example state equation: $Q(t+1) = \overline{Q(t)}x + Q(t)y$, where x & y are inputs, Q(t) is the current state, and Q(t+1) is the next state
- Often the function defining the output of a sequential circuit included in the list of state equations.

Find state equation

- 3 Steps:
  - Step 1: Find **inputs to flip flops** (J, K, T, …) in terms of **current states** of flip flops (Q_A, Q_B, …) and **inputs to circuit** (x, y, …)
  - Step 2: Find the **next states** of flip flops in terms of **flip flop inputs** and **current state**, e.g. find $Q_A(t+1)$ in terms of J,K for JK flip flop; $Q_B(t+1)$ in terms of T for T flip flop. Equivalent to characteristic functions for given flip flops.
  - Step 3: Substitute Step1 into Step 2 to get state equations.

State equation example

- Find the state equation of this circuit:



(a) Circuit diagram

1. Find inputs to flip flop (D) in terms of current state of flip flop (Q(t)) and inputs (x, y)

$$D = x \oplus y \oplus Q(t)$$

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (D)

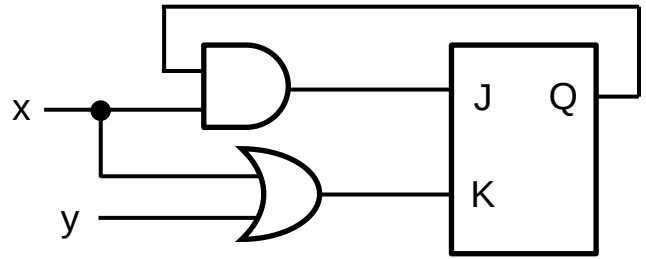
$$Q(t+1) = D$$

3. Substitute 1 into 2.

$$Q(t+1) = x \oplus y \oplus Q(t)$$

State equation example

- Find the state equation of:



Steps:

1. Find inputs to flip flop (J,K) in terms of current state of flip flop (Q(t)) and inputs (x, y)

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (J,K)

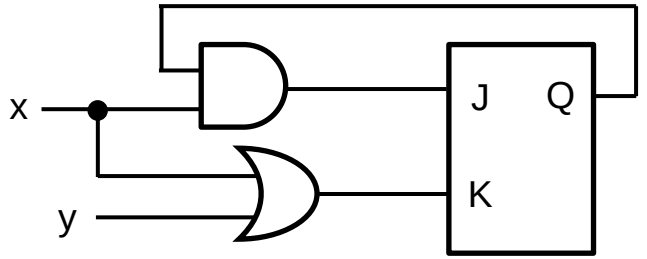
3. Substitute

State equation example (step 1)

1. Find inputs to flip flop (J,K) in terms of current state of flip flop (Q(t)) and inputs (x, y)

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (J,K)

3. Substitute



$$J = xQ(t)$$
$$K = x + y$$

State equation example (step 2)

1. Find inputs to flip flop (J,K) in terms of current state of flip flop (Q(t)) and inputs (x, y)

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (J,K)

3. Substitute

more complicated than D Flip flop. Need to find characteristic equation of JK flip flop for complete relationship of Q(t+1) and J and K.

J	K	Q	Q+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Q+

J \ KQ	00	01	11	10
0	0	1	0	0
1	1	1	0	1

$$\mathbf{Q(t+1) = J\overline{Q(t)} + \bar{K}Q(t)}$$

Characteristic equation

State equation example (step 3/all steps)

1. Find inputs to flip flop (J,K) in terms of current state of flip flop (Q(t)) and inputs (x, y)

$$J = xQ(t)$$
$$K = x + y$$

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (J,K)

$$Q(t+1) = J\overline{Q(t)} + \overline{K}Q(t)$$

(Characteristic equation)

3. Substitute

$$Q(t+1) = xQ(t)\,\overline{Q(t)} + \overline{x+y}\,Q(t)$$
$$= \overline{x+y}\,Q(t)$$

$$Q(t+1) = \overline{x+y}\,Q(t)$$

(State equation)

Characteristic Equations of all FFs

Characteristic equations of an FF is defined as the state equation of the given flip flop.
i.e. the **next state** in terms of the **current state** and the **inputs** to the flip flop

SR Flip Flops

$$Q(t+1) = S + \bar{R}Q(t)$$

D Flip Flops

$$Q(t+1) = D$$

Know these or know how to derive them!

JK Flip Flops

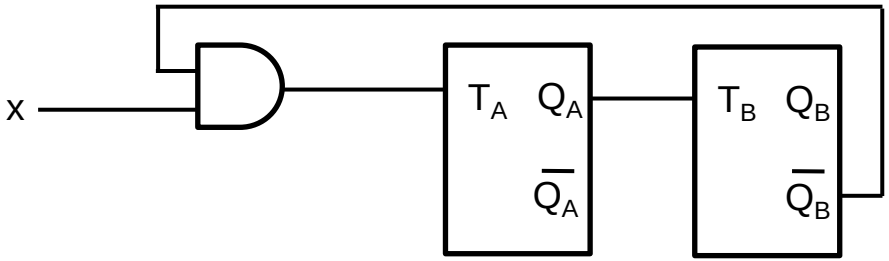
$$Q(t+1) = J\overline{Q(t)} + \bar{K}Q(t)$$

T Flip Flops

$$Q(t+1) = \bar{T}Q(t) + T\overline{Q(t)}$$
$$= T \oplus Q(t)$$

State equation example

- Find the state equations of:



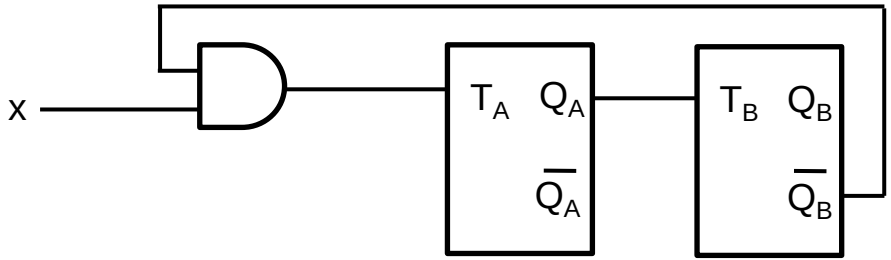
Steps:

1. Find inputs to flip flop (T) in terms of current state of flip flop (Q(t)) and circuit inputs (x, y)

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (T)

3. Substitute

State equation example (cont.)



1. Find inputs to flip flop (T) in terms of current state of flip flop (Q(t)) and circuit inputs (x, y)

$$\mathbf{T_A = xQ_B(t)}$$
$$\mathbf{T_B = Q_A(t)}$$

2. Find next state of flip flop (Q(t+1)) in terms of input to flip flop (T)

$$\mathbf{Q_A(t+1) = T_A \oplus Q_A(t)}$$
$$\mathbf{Q_B(t+1) = T_B \oplus Q_B(t)}$$

3. Substitute

$$\mathbf{Q_A(t+1) = xQ_B(t) \oplus Q_A(t)}$$
$$\mathbf{Q_B(t+1) = Q_A(t) \oplus Q_B(t)}$$

Note there are 2 state equations for 2 flip flops

State Table

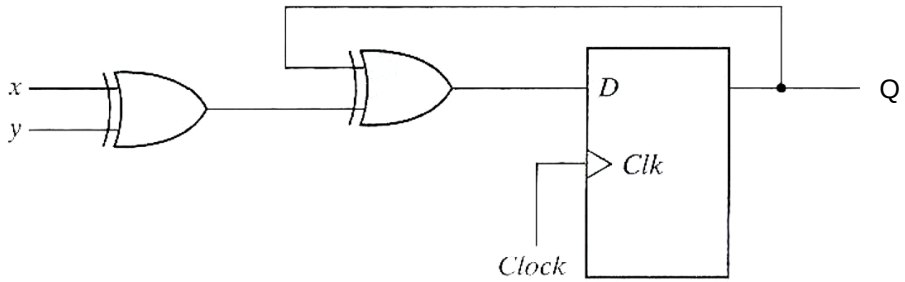
- A table describing the behavior of a sequential circuit as a function of stable internal conditions – **states** – and **input variables**. For each combination of these, the **next state** of the circuit is specified together with any **output variables**.
- If no inputs or outputs exist, leave out their respective columns

Input(s)	Current states		Next states		Output(s)
x	Q_A	Q_B	Q_A+	Q_B+	y
0	0	0	0	1	0
0	0	1	1	1	1
…					
1	1	1	1	0	1

Example

- Find the state table of the circuit:
  1. Find the state equation
  2. Plug in values to find the state table

$$Q(t+1) = x \oplus y \oplus Q(t)$$

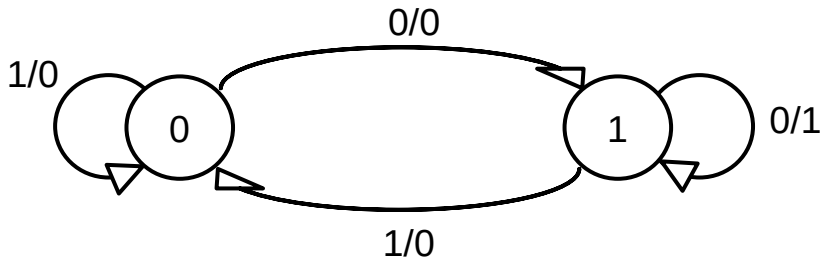


(a) Circuit diagram

x	y	Q	Q+
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

State Diagram

- The information available in a **state table** can be represented graphically in the form of a **state diagram**.
- In this type of diagram, a state is represented by a circle, and the transitions between states are indicated by directed lines connecting the circles.
- Inputs and outputs are assigned to each arrow in the format: “input/output”
- Example:

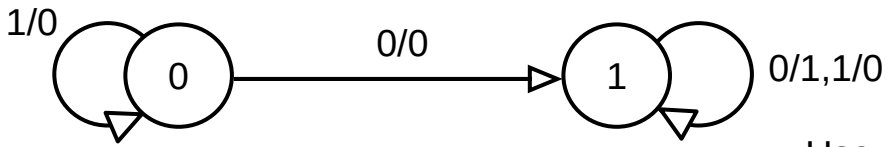


Example
1/0: Input of 1,
Output of 0

State diagram example

Find the state diagram of the state table:

x	Q	Q+	y
0	0	1	0
0	1	1	1
1	0	0	1
1	1	1	0

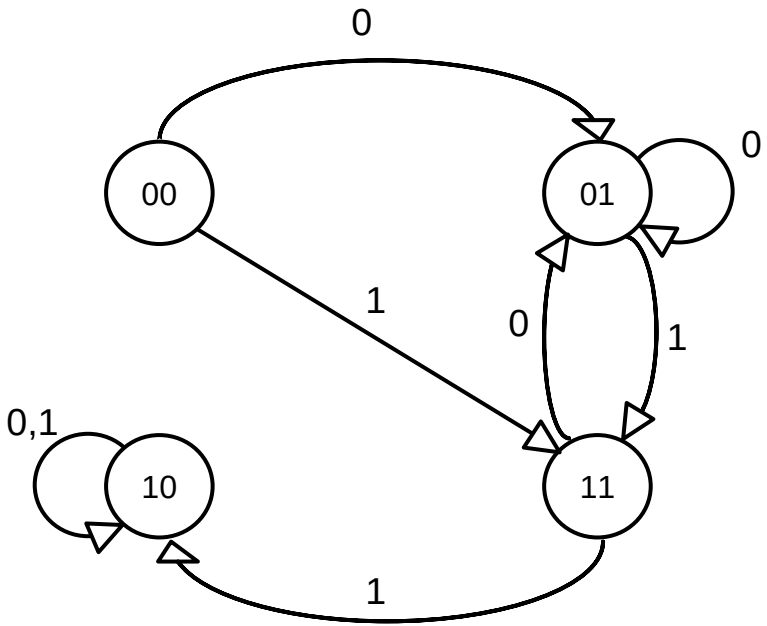


Use commas to separate shared values on the same path

State diagram example

Find the state diagram of the state table:

x	Q_A	Q_B	Q_A+	Q_B+
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0



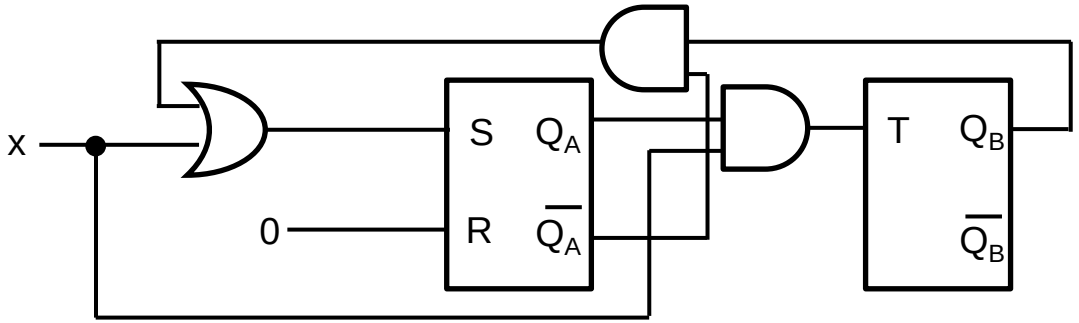
For more than one state bit, you have to write a key showing what the order of the Qs are within each state bubble

State key is Q_AQ_B,
so for Q_A=0 and Q_B=1, the state is 01

State diagram example

Find the state diagram of the circuit:

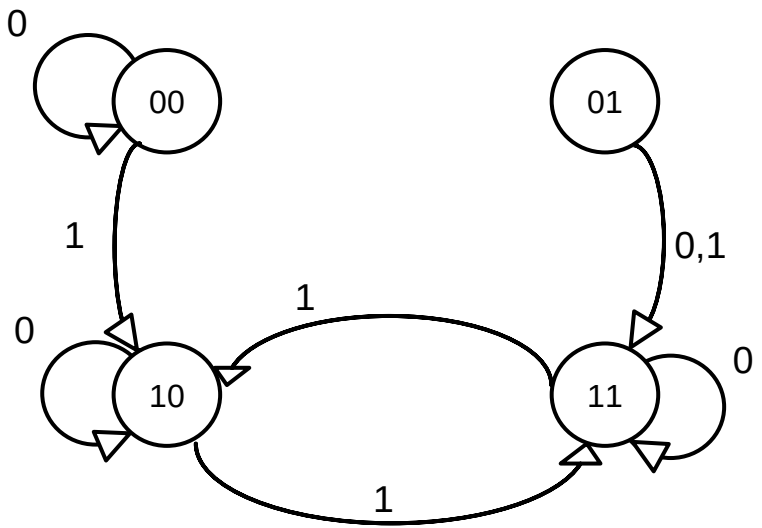
1. Find the state equations
2. Find the state table
3. Draw the state diagram



State diagram example, cont.

$$Q_A(t+1) = x + Q_B(t)\overline{Q_A(t)} + Q_A(t)$$
$$Q_B(t+1) = xQ_A(t)\overline{Q_B(t)} + \overline{xQ_A(t)}Q_B(t)$$

x	Q_A	Q_B	Q_A+	Q_B+
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0



Summary

- State Equation
- State Table
- State Diagram

References

- https://redirect.cs.umbc.edu/courses/undergraduate/CMSC313/Fall03/cpatel2/slides/slides19.pdf