UMBC

Number Systems

CMSC 313: Assembly Language and Computer Organization

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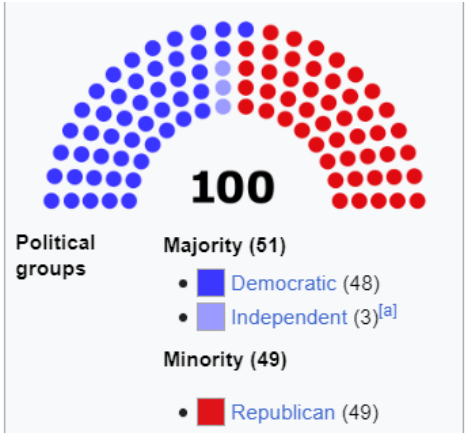
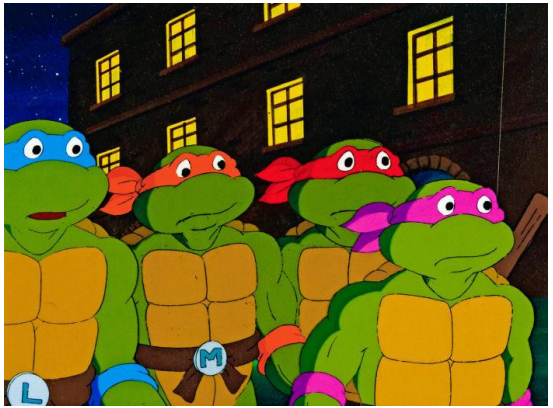
Number Systems

- Radix: the base used in the numeric system
- Ex: “100” can be this many:



In base 10 (decimal)

In base 8 (octal)



In base 2 (binary)

General Definition

- In a system with radix b where $b > 1$, a string of digits $d_n \ldots d_1$ denotes the number

$$d_n b^{n-1} + d_{n-1} b^{n-2} + \ldots + d_2 b^1 + d_1 b^0$$

  where $0 \leq d_i < b$.

- Notation: <Number>$_{\text{<base>}}$
- Ex: 101 in base-2 is 101_2
- $2{,}384_{10} = 2\text{x}10^3 + 3\text{x}10^2 + 8\text{x}10^1 + 4\text{x}10^0$

Convert binary (base 2) to decimal:

- $101_2 = ?_{10}$
- $= 1x2^2 + 0x2^1 + 1x2^0$

  $= 5_{10}$

Convert decimal to binary:

- $115_{10} = ?_2$

```
115 ÷ 2 = 57 Remainder 1  <- Least significant bit (LSB)
 57 ÷ 2 = 28 Remainder 1
 28 ÷ 2 = 14 Remainder 0
 14 ÷ 2 = 7  Remainder 0
  7 ÷ 2 = 3  Remainder 1
  3 ÷ 2 = 1  Remainder 1
  1 ÷ 2 = 0  Remainder 1  <- Most significant bit (MSB)
```

1110011_2

Base 16 (Hexadecimal)

- 0,1,2,3,4,5,6,7,8,9, …
- What comes next?
- A,B,C,D,E,F
- What comes next?
- 10,11,12, … 19,
- What comes next?
- 1A, 1B, 1C, 1D, 1E, 1F
- What comes next?
- 20, 21, 22, …

Convert Hexadecimal to decimal

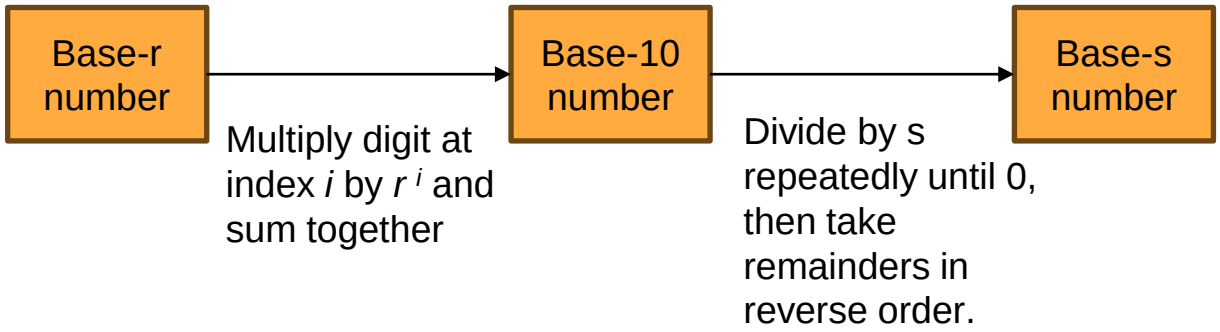
- $3E7_{16}$ = 0x3E7 = $?_{10}$

$$
\begin{aligned}
& 3\text{x}16^2 + \text{E}\text{x}16^1 + 7\text{x}16^0 \\
= \; & 3\text{x}16^2 + 14\text{x}16^1 + 7\text{x}16^0 \\
= \; & 768 + 224 + 7 \\
= \; & 999
\end{aligned}
$$

Convert base-r to base-s

Steps:

1. Convert base-r to base-10
2. Convert base-10 to base-s



Hex to Binary

Convert $82B_{16}$ to base 2

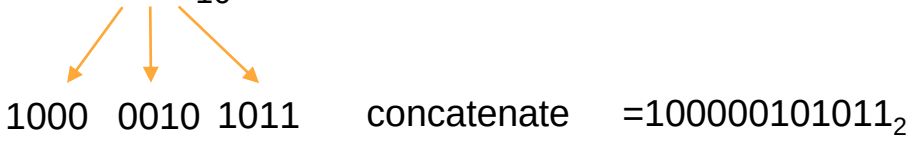
$$82B_{16} = 8x16^2 + 2x16^1 + Bx16^0$$
$$= 8x16^2 + 2x16^1 + 11x16^0$$
$$= 2048 + 32 + 11$$
$$= 2091_{10}$$

```
2091 ÷ 2 = 1045 Remainer 1 (LSB)
1045 ÷ 2 = 522  Remainer 1
522  ÷ 2 = 261  Remainer 0
261  ÷ 2 = 130  Remainer 1
130  ÷ 2 = 65   Remainer 0
65   ÷ 2 = 32   Remainer 1
32   ÷ 2 = 16   Remainer 0
16   ÷ 2 = 8    Remainer 0
8    ÷ 2 = 4    Remainer 0
4    ÷ 2 = 2    Remainer 0
2    ÷ 2 = 1    Remainer 0
1    ÷ 2 = 0    Remainer 1 (MSB)
```

$1000\ 0010\ 1011_2$

Shortcut

- If *r* is a power of *s* or *s* is a power of *r*, you do not need to convert through base 10.
- Split each digit of *r* and expand/contract individually to base *s*.
- Example: r = 16, s = 2. Convert $82B_{16}$ to base 2



- Example: r = 16, s = 2. Convert 010111111010_2 to base 16

0101|1111|1010
5 F A
append = 5FA

Why the Shortcut works

$A3D_{16} = 10x16^2 + 3x16^1 + 13x16^0$

$= (1x2^3 + 0x2^2 + 1x2^1 + 0x2^0)x16^2 +$
$(0x2^3 + 0x2^2 + 1x2^1 + 1x2^0)x16^1 +$
$(1x2^3 + 1x2^2 + 0x2^1 + 1x2^0)$

$= (1x2^3 + 0x2^2 + 1x2^1 + 0x2^0)x2^8 +$
$(0x2^3 + 0x2^2 + 1x2^1 + 1x2^0)x2^4 +$
$(1x2^3 + 1x2^2 + 0x2^1 + 1x2^0)$, by rewriting 16 as a power of 2.

$= 1x2^{11} + 0x2^{10} + 1x2^9 + 0x2^8 +$
$0x2^7 + 0x2^6 + 1x2^5 + 1x2^4 +$
$1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 = 1010001111012.$

A joke

There are 10 kinds of people in this world

- Those who understand binary
- Those who don’t
- Those who didn’t expect this joke to be in base 3

Definition extension for fractional numbers

- In a system with radix b where $b > 1$, a string of digits

Decimal place

$$d_1 d_2 \ldots d_{n-1} d_n . d_{n+1} d_{n+2} \ldots d_{n+m-1} d_{n+m}$$

denotes the number

$$\underbrace{d_1 b^{n-1} + d_2 b^{n-2} + \ldots + d_{n-1} b^1 + d_n b^0}_{\text{characteristic}} + \underbrace{d_{n+1} b^{-1} + d_{n+2} b^{-2} \ldots + d_{n+m-1} b^{m-1} + d_{n+m} b^m}_{\text{mantissa}}$$

where $0 \leq d_i < b$.

- *integer-part (characteristic)* can be converted separate from the *fractional-part (mantissa)*

Between 0 and 1 – binary to decimal

- Convert 100.1101_2 to decimal

- $100_2 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$
  $= 4$

- $0.1101_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$
  $= 0.5 + 0.25 + 0 + 0.0625$
  $= 0.8125$

- $4 + 0.8125 = 4.8125_{10}$

Between 0 and 1 – decimal to binary

- Convert 14.8125_{10} to base 3

```
14 ÷ 3 = 4 Remainder 2 LSB
 4 ÷ 3 = 1 Remainder 1
 1 ÷ 3 = 0 Remainder 1 MSB
```

Multiply

```
0.8125 x 3 = 2.4375 | 2 MSB
0.4375 x 3 = 1.3125 | 1
0.3125 x 3 = 0.9375 | 0
0.9375 x 3 = 2.8125 | 2 LSB
0.8125 x 3 = 2.4375 duplicate
```

$$112.\overline{2102}_3$$

Summary

- Convert base-r to base-10
  - Splitting digits and multiplying by the value of their place
- Convert base-10 to base-r
  - Dividing for characteristic (integer part) and multiplying for mantissa (fractional part)
- Convert base-r to base-s by going through base-10
- Convert base-r to base-s with shortcut if r is a power of s or vice versa
- Convert fractional parts of number

References

- Nick Allgood's slides
- https://www.cs.cornell.edu/courses/cs3410/2018fa/schedule/slides/03-numbers-and-arithmetic.pdf
- https://www.cs.ucf.edu/courses/cop3502/fall2011/Lectures/Lec7_BaseConversion.pdf